HEAT TRANSFER IN TURBULENT NATURAL CONVECTION ON A VERTICAL PERMEABLE SURFACE

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 3, pp. 143-147, 1968

ABSTRACT: A theoretical and experimental study is presented for heat transfer in turbulent natural convection on vertical surfaces with uniform and homogeneous air injection and withdrawal.

There are several papers [1-4] on heat transfer in laminar free convection on a vertical surface with pore injection and withdrawal, but there are no published studies of heat transfer in turbulent natural convection on permeable vertical surfaces.

1. In theoretical discussion we assume as first approximation that the tangential stress and the heat flux at the wall are determined as for turbulent natural convection on impermeable surfaces.

In the second approximation we take into account in the laws of heat flux and tangential stress the effects of wall permeability on one of the theoretical solutions derived for a turbulent boundary layer in forced flow around a planar permeable surface.

Finally, the method of relative correspondence, as applied to a laminar boundary layer [3], is applied to a turbulent boundary layer, since this method most rapidly leads to the final results.

We can give the following forms to the integral equations for the momentum and energy in natural convection on a vertical permeable surface with uniform injection and withdrawal:

$$\frac{d}{dx}\int_0^\delta u^2 dy = g\beta\int_0^\delta \theta dy - \frac{\tau_w}{\rho_\infty},$$

$$\frac{d}{dx}\int_0^\delta \theta u dy = \frac{q_w}{c_p g \rho_\infty} + v_w \rho \theta_w \qquad (1.1)$$

Here u is the longitudinal component of the velocity in the boundary layer, v_w is the velocity of injection (withdrawal), τ_w is the tangential stress at the wall, q_w is the heat flux at the wall, t is temperature, $\rho = \rho_w/\rho_\infty$ (ratio of density at the wall to density far from the heated surface), c_p is the specific heat at constant pressure, β is the bulk expansion coefficient, δ is the thickness of the boundary layer, t_w is surface temperature, t_∞ is the temperature of the unperturbed medium, and $\theta = t - t_\infty$, $\theta_w = t_w - t_\infty$.

In these equations we have neglected energy dissipation and have taken the physical properties of the fluid as constant, apart from the density in the term for the lifting force.

Consider an isothermal surface through which a fluid is uniformly injected or withdrawn. The fluid has the properties of the unperturbed medium; i.e., v_w is not dependent on the longitudinal coordinate x.

The 1/7 law is applied to the distribution of u and θ in the turbulent boundary layer:

$$u = u_1\left(\frac{y}{\delta}\right)^{1/7}\left(1-\frac{y}{\delta}\right)^4,$$

$$\theta = \theta_w\left[1-\left(\frac{y}{\delta}\right)^{1/7}\right]. \qquad (1.2)$$

2. First approximation. We take τ_w and q_w to be as for turbulent natural convection on a vertical impermeable surface [5]:

$$\tau_w = 0,253\,\rho_w u_1^2\left(\frac{\nu}{u_1\delta}\right)^{1/2},$$

$$q_w = 0,253\, g\rho_w c_p u_1\left(\frac{\nu}{u_1\delta}\right)^{1/2} P^{-2/3}\theta_w. \qquad (2.1)$$

The solution of (1.1) subject to (1.2)-(2.1) gives the local Nusselt number for suction and moderate injection:

$$\frac{N}{N_0} = 0.795\,\frac{\varkappa}{\rho^{1/3}}$$

$$\varkappa = \left\{\left[\left(1-\frac{1}{2}\eta\right)+(1-\eta)^{1/2}\right]^{1/3} + \left[\left(1-\frac{1}{2}\eta\right)-(1-\eta)^{1/2}\right]^{1/3} - 0.795\,\eta^{1/3}\right\} \qquad (2.2)$$

The injection (suction) parameter η in these formulas is

$$\eta = 182\,\rho\,\frac{R^3}{G}\left(\frac{P^{2/3}}{2.14+P^{2/3}}\right)^2. \qquad (2.3)$$

In (2.1)-(2.3) P, R, and G are the Prandtl, Reynolds, and Grashof numbers

$$P = \frac{\nu}{a}, \qquad R = \frac{v_w x}{\nu}, \qquad G = \frac{g\beta\theta_w x^3}{\nu^2},$$

while N_0 is the Nusselt number for turbulent natural convection on an impermeable vertical surface, given by [5]

$$N_0 = \frac{0.2\,(GP^{5/3})^{1/3}}{(2.14+P^{2/3})^{1/3}}. \qquad (2.4)$$

It follows from (2.2) that η should not exceed one for moderate injection. The solution to (1.1) for strong injection ($\eta > 1$) is

$$\frac{N}{N_0} = 1.23\,\frac{\eta^{1/3}}{\rho^{1/3}}\left[\cos\left(\frac{\pi}{3}-\frac{\varepsilon}{3}\right)-\frac{1}{2}\right],$$

$$\cos\varepsilon = \left(1-\frac{2}{\eta}\right). \qquad (2.5)$$

For air (P = 0.72) we should take as follows in (2.2)-(2.5):

$$N_0 = 0.13\,(GP)^{1/3}, \quad \eta = 13.6\,\rho^{1/3}\,R^3/G. \qquad (2.6)$$

3. We incorporate the permeability in the laws for τ_w and q_w, which should allow one to estimate more precisely the effects of injection and suction on the heat transfer in turbulent natural convection. For this purpose we introduce into (2.1) corrections for the permeability from a solution obtained for a turbulent boundary layer on a permeable planar surface with forced flow. As that solution we take a theoretical result [6] for turbulent heat transfer in injecting a homogeneous gas into a forced-convection flow:

$$\frac{c_f}{c_{f0}} = \frac{q_w}{q_{w0}} = \left(1-K\rho\,\frac{v_w}{u_1 c_f}\right)^2. \qquad (3.1)$$

Coefficient K varies in value with the R of the incident flow; in particular, K = 0.25 for $R \to \infty$. The solution to (3.1) agrees well with results [7,8] on porous injection and suction in a turbulent boundary layer in forced flow.

The minimum G in turbulent natural convection may be taken as 10^8, which corresponds [5] to $R = 0.75 \cdot 10^4$ for P = 0.72, this being derived for the maximum velocity in a turbulent boundary layer produced by natural convection on a vertical surface. Then K varies in the range 0.2 to 0.25 [6]. It has been shown [3] that there is a general analogy between laminar natural convection and forced laminar flow on a permeable surface. We expect such an analogy also for a turbulent layer, but we expect also a certain quantitative discrepancy because of the term containing buoyancy forces in the equation of motion.

Then we introduce a permeability correction [6] into (2.1):

$$\tau_w = 0.253\,\rho_w u_1^2\left(\frac{\nu}{u_1\cdot\delta}\right)^{1/2}\left[1-1.975\,K\rho v_w\left(\frac{\nu u_1}{\delta}\right)^{-1/2}\right]^2,$$

Experimental Results on the Effects on Heat Transfer from Injection and Withdrawal of Air in Turbulent Natural Convection on Vertical Surfaces

	x, m	t_w, °C	t_∞, °C	$v_w\cdot10^3$, m/sec	R	$G\cdot10^9$	P	η	$\frac{N}{N_0}$
Injection	1.27	74.2	26.2	0	0	9.2	0.864	0.0	1.0
				9.07	636.0			0.302	0.47
				10.3	726.6			0.476	0.42
				11.57	816.1			0.68	0.39
Injection	1.37	74.2	26.2	0	0	11.5	0.864	0.0	1.0
				9.07	687.0			0.326	0.46
				10.3	783.8			0.476	0.43
				11.57	816.1			0.544	0.37
Injection	1.27	58.0	26.0	0	0	6.91	0.906	0.0	1.0
				2.31	170.5			0.009	0.78
				3.85	284.0			0.039	0.67
				6.13	452.0			0.163	0.52
				6.94	512.0			0.238	0.48
				8.48	626.0			0.431	0.44
				10.04	741.0			0.722	0.39
				11.57	825.2			1.00	0.36
				13.1	934.0			1.44	0.35
				14.81	1056			2.09	0.32
				16.2	1155			2.74	0.29
Injection	1.37	58.0	26.0	0	0	8.69	0.906	0.0	1.0
				2.31	184.0			0.008	0.75
				3.85	306.0			0.04	0.66
				6.13	488.0			0.163	0.49
				6.94	553.0			0.239	0.44
				8.48	675.0			0.431	0.41
				10.04	799.0			0.714	0.37
				11.57	890.0			0.99	0.33
				13.1	1008			1.429	0.31
				14.81	1139			2.093	0.29
				16.2	1246			2.735	0.28
Suction	1.27	42.0	26.2	0	0	3.91	0.949	0.0	1.0
				1.48	115.6			0.005	1.39
				2.59	202.4			0.027	1.68
				3.88	303.4			0.091	1.94
				5.55	433.7			0.258	2.04
				6.94	542.0			0.516	2.37
				8.79	686.7			1.065	2.46
				10.18	795.0			1.653	2.53
				11.57	904.0			2.69	2.58
Suction	1.37	42.0	26.2	0	0	4.91	0.949	0.0	1.0
				1.48	124.7			0.003	1.35
				2.59	218.3			0.026	1.67
				3.88	327.0			0.091	2.00
				5.55	467.8			0.258	2.35
				6.94	585.0			0.525	2.38
				8.79	740.9			1.066	2.50
				10.18	857.8			1.651	2.66
				11.57	975.3			2.42	2.70
Suction	1.27	71.0	26.8	0	0	8.52	0.872	0.0	1.0
				2.59	182.7			0.007	1.57
				3.88	273.7			0.028	1.73
				5.78	407.7			0.093	1.93
				6.94	491.6			0.164	2.18
				8.79	620.0			0.33	2.30
				10.0	717.8			0.509	2.35
				11.57	829.9			0.793	2.43
				12.97	923.0			1.09	2.50
				14.35	1029			1.504	2.54
				16.2	1162			2.18	2.61
				17.75	1271			2.844	2.62
Suction	1.37	71 0	26.8	0	0	10.69	0.872	0.0	1.0
				2.59	197.1			0.008	1.53
				3.88	295.2			0.028	1.76
				5.78	439.4			0.093	1.97
				6.94	530.0			0.164	2.22
				8.79	668.8			0.33	2.37
				10.0	774.0			0.513	2.46
				11.57	896.4			0.889	2.52
				12.97	1002			1.11	2.55
				14.35	1110			1.512	2.55
				16.2	1253			2.18	2.63
				17.75	1372			2.855	2.62

$$q_w = 0.253\, g\rho_w c_p u_1 \left(\frac{\nu}{u_1\cdot\delta}\right)^{1/2} P^{-2/3}\theta_w \times$$

$$\times\left[1 - 1.975\, K\rho v_w \left(\frac{\nu u_1}{\delta}\right)^{-1/2}\right]^2. \qquad (3.2)$$

Here we have omitted the subscript 0 to u_1 and δ because it is assumed that here these will be the characteristic velocity and the boun boundary-layer thickness for a permeable surface.

Solution of (1.1) with (1.2) and (3.2) gives

$$\frac{N}{N_0} = 0.795\frac{\varkappa}{P^{1/3}}\left[1 - 0.555\, K\, \frac{P\eta^{1/3}}{\varkappa}\left(\frac{P^{2/3}+2.14}{P^{1/3}}\right)\right]^2, \qquad (3.3)$$

where $\varkappa$ is a function of η and P.

For air (P = 0.72) we have

$$\frac{N}{N_0} = 0.795\frac{\varkappa}{P^{1/3}}\left[1 - 2.03\, K\, \frac{P\eta^{1/3}}{\varkappa}\right]^2. \qquad (3.4)$$

Experiment shows that the correction factor for air may be taken as K = 0.125.

Figure 1 gives curves for $\varkappa(\eta)$ for air with $-3 < \eta < 3$ to facilitate calculations from (3.4); curve 1 is for suction and curve 2 is for injection.

4. The method of relative correspondence [4] has been used to investigate heat transfer in laminar natural convection on a permeable

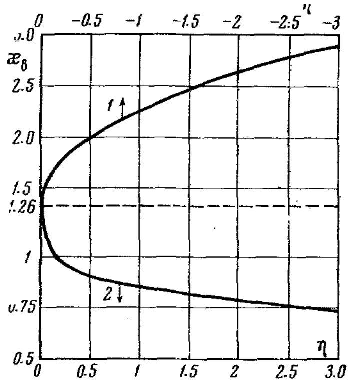

Fig. 1

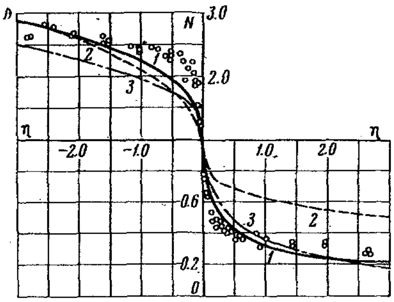

Fig. 2

surface, and this gives the final results most rapidly. Here it is assumed that there is a general analogy between forced flow and natural convection in the layer of liquid directly adjoining the wall. We use the solution for heat transfer in forced motion on a permeable planar surface.

As that solution we take (3.1) for the limiting case $R \to \infty$ in the form

$$\frac{N}{N_0} = \left(1 - 0.25\,\rho\,\frac{v_w}{u_{1_0} c_{f_0}}\right)^2, \tag{4.1}$$

where N_0 is defined by (2.4). The value of $u_{10}c_{f0}$ is defined by the solution for turbulent natural convection on an impermeable vertical surface [5], which gives

$$u_{1_0} c_{f_0} = 0.4\,\frac{\nu}{x}\,\frac{(GP^{2/3})^{1/3}}{(2.14 + P^{2/3})^{1/3}}. \tag{4.2}$$

Substitution of (4.2) into (4.1) gives the final formula for the heat transfer:

$$\frac{N}{N_0} = \left[1 - 0.1105\,\rho^{2/3}\eta^{1/3}\,\frac{2.14 + P^{2/3}}{P^{2/3}}\right]^2. \tag{4.3}$$

This formula takes the following form for P = 0.72 (air):

$$\frac{N}{N_0} = (1 - 0.405\,\rho^{2/3}\eta^{1/3})^2. \tag{4.4}$$

5. The experiments were performed with an apparatus modified from that of [4]. Grashof numbers in excess of the upper limit for the transition zone were provided by a heated preceding section 1 m high. The heat-transfer coefficient was found by interpreting interference patterns. The maximum error in determining the air flow rate was 4%. The wall temperature deduced from the interference patterns agreed to 0.3% with that recorded by thermocouples. The ranges in the principal quantities were as follows:

$$G = (3.5 - 11.5)\,10^9,\quad R = (-1.4\text{--}1.4)\,10^3,\quad \eta = -2.85\text{--}3.7$$

$$\rho = 0.864\text{--}0.95,\quad P = 0.72$$

The results are shown in the table. The mean temperature of the boundary layer was used as the defining temperature in processing the results.

Figure 2 compares the solutions with experiment for heat transfer in turbulent natural convection on a permeable vertical surface with uniform injection and withdrawal of air ($P = 0.72$, $\rho = 1.0$, $K = 0.125$). Curve 1 is from (3.4), curve 2 from (2.2), curve 3 from (4.4); the points represent experiment.

In the injection region, (3.4) gives the best agreement with experiment if we use $K = 0.125$ and the solution from relative correspondence.

There is very little effect from the permeability in the suction region on the tangential stress and heat flux.

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